数论日会议手册

会议时间：2024 年 12 月 27 日周五 9:00-16:00

参会方式：腾讯会议-880360912

组 织 人：方江学，王浩然，蔡立

会议日程

时间	报告内容	主持人
09:00 - 09:30	Analytic views of Goldfeld's conjecture on quadratic twists of elliptic curves 翟帅 山东大学	许宾
09:35 - 10:05	Local theta correspondence and local periods 张翀 南京大学	翟帅
10:10 - 10:40	On local converse theorems 张庆 华中科技大学	张翀
10:45 - 11:15	Nonvanishing of quadratic twist L-values 熊玮 湖南大学	张庆
13:30 - 14:00	Signed Euler positivity via p-adic spectral theory 张鼎新 清华大学	熊玮
14:05 - 14:35	Obstructions to local-global principle on algebraic stacks 吕昌 中国科学院信息工程研究所	张鼎新
14:40 - 15:10	A variant of Chebatorev density theorem and its applications in cryptography 丁治国 中南大学	吕昌
15:15 - 15:45	Polynomials over finite fields with small value sets and monodromy groups 许宾 四川大学	丁治国

报告摘要

Analytic views of Goldfeld's conjecture on quadratic twists of elliptic curves
翟帅 山东大学

In 1979, Goldfeld conjectured that, for every elliptic curve defined over the rationals, amongst the set of all its quadratic twists with root number +1, there is a subset of density one where the central L-value of the twist is non-zero, and amongst the set of all its quadratic twists with root number -1, there is a subset of density one where the central

L-value of the twist has a zero of order equal to 1. In the past few years, some remarkable progress has been made in this direction, e.g., the work of Smith and Burungale--Tian. In this presentation, I will primarily explore the analytic methods attacking Goldfeld's conjecture.

Local theta correspondence and local periods

张翀 南京大学

Theta correspondence can be used to study the relation between certain periods of the representations of the reductive dual pairs. This approach has been vastly studied by many mathematicians, especially by Dipendra Prasad, Wee Teck Gan, their collaborators and students. I will report some recent advances on this topic, focusing on the case of Galois periods.

On local converse theorems

张庆 华中科技大学

Let F be a local field and G be a quasi-split reductive group over F. The local converse problems asks whether a (generic irreducible smooth) representation of $G(F)$ can be uniquely determined by its twisted gamma factors. In this talk, I will report certain local converse theorems for quasi-split classical groups, which include the symplectic groups and unitary groups.

Nonvanishing of quadratic twist L-values

熊玮 湖南大学

In 1982, Waldspurger showed that for any irreducible cuspidal automorphic representation of PGL(2) over a number field with epsilon factor +1, there always exists a quadratic twist of this representaton such that the central L-value of this quadratic twist is nonvanishing. The proof used theta lifting and Flicker's correspondence between automorphic representations of the double cover of GL(2) and those of GL(2). In 1987 Jacquet reproved this result by relative trace formula, in 1995 Friedberg and Hoffstein generalized this result to more representations of GL(2) by studying some Dirichlet series, and recently Liu and Xu gave a new apporach to this result by using the automorphic decent method . In this talk I will discuss the original proof by Waldspurger.

Signed Euler positivity via p-adic spectral theory

张鼎新 清华大学

A very affine variety is a variety that can be embedded into an algebraic torus. Sabbah (over

the complex numbers) and Gabber-Loeser (over any field) showed that if X is a nonsingular very affine variety of dimension n, then $(-1)^n \chi(X) \geqslant 0$. I will provide an elementary proof of this result using p-adic functional analysis and demonstrate how this straightforward approach leads to various generalizations.

Obstructions to local-global principle on algebraic stacks

吕昌 中国科学院信息工程研究所

I will talk about obstructions such as Brauer-Manin, descent, finite-descent on algebraic stacks by extending some classical results such as the exact sequence of Sansuc, torsionness of Brauer groups, properties for Brauer-Manin sets, and relations between some obstructions. Time permitting, I will introduce a new approach to relations using cohomological descent. Part of these works is joint with Han Wu.

A variant of Chebatorev density theorem and its applications in cryptography.

丁治国 中南大学

We give a variant of the Chebotarev density theorem for a certain class of polynomials over finite fields with varying coefficients. We apply this to connect the Niho problem of cross correlation of m-sequences in cryptography with the determination of arithmetic and geometric Galois groups of certain finite etale morphisms of varieties over finite fields. We then solve the Niho problem by computing the relevant Galois groups. This is joint work with Michael Zieve.

Polynomials over finite fields with small value sets and monodromy groups

许宾 四川大学

Polynomials over finite fields are extensively studied due to various motivations. One way to classify these polynomials is by examining their value sets. For instance, polynomials whose value sets encompass the entire base field are known as permutation polynomials, which have many important applications in mathematics and related fields. In this talk, we focus on analyzing the value sets of polynomials over finite fields through their monodromy groups (specific associated Galois groups). We will begin by reviewing some background and previous work, and then present results and further problems for polynomials with small value sets obtained via this approach.